PRELIMINARY COMMUNICATION

High resolution solid-state N.M.R.

Averaging of second-order effects by means of a double-rotor†

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By means of rotations around two axes inclined at zeros (magic angles) of
the \( l = 2 \) and \( l = 4 \) Legendre polynomials, first-order and second-order N.M.R.
broading can be averaged away. Experiments with a double-rotor on the
central \( (1/2 \leftrightarrow -1/2) \) transition of sodium-23 in polycrystalline sodium oxalate
illustrate the elimination of broadening due to second-order quadrupolar effects
leading to a thirtyfold increase in resolution compared to magic-angle spinning.

Spectral broadening due to anisotropic second-order frequency shifts has been
one of the serious obstacles to high resolution N.M.R. of quadrupolar nuclei in the
solid state. One now understands that neither rotation of the sample around any
one axis, as in (say) magic-angle spinning (MAS) [1], nor radiofrequency manipula-
tions of the spins [2] can eliminate such effects of second-order spatial anisotropy. It
is therefore commonly surmised that the only solutions to this problem are either to
perform MAS experiments in higher magnetic fields (since second-order effects are
inversely proportional to the field strength) [3], or to remove the magnetic field and
recover a scalar hamiltonian as in zero-field N.M.R. [4].

There is, however, an alternative approach [5, 6], one version of which we
demonstrate experimentally in this communication, namely to rotate the sample
around more than one axis. To appreciate why this is so and how it can be done,
consider the arrangement in figure 1, where the rotor containing the sample is itself
suspended inside a second rotor. Recall that the spatial anisotropy of secular
nuclear interactions is conveniently described by a sum of spherical harmonics of
increasing rank \( l \). In high field, the leading terms in the orientation-dependent

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N.M.R. frequencies

\[ \omega = \omega_0 + \sum_{l} \sum_{m=-l}^{+l} A_{lm} Y_{lm}(\Omega), \]  

are of rank \( l = 2 \) and 4, where \( Y_{lm}(\Omega) \) are spherical harmonics (for non-secular terms an expansion in full Wigner matrices \( D^{(0)} \) is necessary), \( \Omega \) denotes the two angles \((\theta, \phi)\) specifying the orientation of the magnetic field in a coordinate system fixed with the sample and the coefficients \( A_{lm} \) depend on the principal values and axes of the spin interaction tensors. First-order effects such as chemical shift anisotropy and secular dipole–dipole couplings are contained in the \( l = 2 \) term, while second-order effects (which arise from correction terms to the average hamiltonian \([2]\)) are expressed by both \( l = 2 \) and \( l = 4 \) terms. The frequency \( \omega_0 \) describes the sum of the isotropic chemical shift and the isotropic part of the high-order couplings.

Figure 1. Schematic diagram of the BETA (Berkeley–Tallinn) double rotor. The angles are \( \theta_1 \) between \( B_0 \) and \( R_1 \), \( \theta_2 \) between \( R_1 \) and \( R_2 \), and \( \theta(t) \) between \( B_0 \) and \( R_2 \). Values of \( \theta_1 \) and \( \theta_2 \) are discussed in the text.

If the sample undergoes reorientation so that \( \Omega \) is made time dependent, then the average frequency from (1) is given by

\[ \langle \omega \rangle = \omega_0 + \sum_{l} \sum_{m=-l}^{+l} A_{lm} \langle Y_{lm}(\Omega) \rangle, \]

where \( \langle \rangle \) denotes average values over the realized values of \( \Omega \). The terms \( \langle Y_{lm}(\Omega) \rangle \) are zero for motion around an axis if the magnetic field assumes at least \( l + 1 \)
equally spaced directions on a cone with half-apex angle \( \theta^{(0)} \) viewed in the sample coordinate system, where \( \theta^{(0)} \) is a zero \( (l\text{th rank magic angle}) \) of the \( l\text{th rank Legendre polynomial} \)

\[
P_l(\cos \theta^{(0)}) = 0.
\]

In the usual case of first-order broadening \((l = 2)\), this corresponds to threefold (or higher) hops [7] or, in the continuous MAS limit, to rotation at the 'normal' magic angle \( \theta \equiv \theta^{(2)} = 54.74^\circ \), leaving an \( l = 4 \) powder pattern with sidebands [8]. Similarly, to remove \( l = 4 \) terms and to leave an \( l = 2 \) powder pattern, the appropriate angle for the axis of rotation is \( \theta^{(4)} = 30.56^\circ \) or \( 70.12^\circ \).

Removal of both \( l = 2 \) and \( l = 4 \) terms calls for a pattern of sample orientations (by no means unique or even minimal) in which each of the three directions of the \( l = 2 \) cone (half-apex angle \( \theta^{(2)} \)) is split into five equally spaced directions of the \( l = 4 \) cone (half-apex angle \( \theta^{(4)} \)). The continuous limit of this polyhedral symmetry [9] is a double-rotation which can be implemented by the arrangement of figure 1. Under a double-rotation with angles \( \theta_1 \) and \( \theta_2 \) (avoiding ratios of the two rotation frequencies near 1, 2, 3 or 4) the average frequency in (2) is given by

\[
\langle \omega \rangle = \omega_0 + \sum_l A_l P_l(\cos \theta_1) P_l(\cos \theta_2),
\]

and the \( l = 2 \) and \( l = 4 \) terms can therefore be removed by choosing \( \theta_1 = \theta^{(2)} \) and \( \theta_2 = \theta^{(4)} \) or \( \theta_1 = \theta^{(4)} \) and \( \theta_2 = \theta^{(2)} \).

The apparatus used in our experiments consists of a DELRIN-MACOR rotor of diameter 5 mm, filled with a powder sample and spinning with gas-lubricated journal bearings inside a VESPEL rotor of 20 mm diameter, maintained by independent gas-lubricated bearings and driven by air jets. The angles used are \( \theta_1 = 54.7^\circ \) and \( \theta_2 = 30.6^\circ \) and the mass distribution is designed to minimize inertial forces on the inner rotor by dynamic balance [9, 10]. Figure 2 shows experimental results for the central \((1/2 \leftrightarrow -1/2)\) transition of sodium-23 in polycrystalline sodium oxalate \((\epsilon^2 Q Q / h \sim 2.5 \text{ MHz}, \eta \sim 0.7)\) at 105 MHz. The top spectrum, from a static sample, exhibits a linewidth of about 12 000 Hz due primarily to dipolar couplings and second-order quadrupolar broadening. This linewidth is reduced to about one-third of its static value under magic-angle spinning in the middle trace, and the lineshape is characterized by the average \( l = 4 \) term of the second-order quadrupolar coupling [8(c)]. Sidebands are apparent at the MAS frequency of \( \sim 4000 \text{ Hz} \). Under double-rotation frequencies of \( \sim 2000 \text{ Hz} \) for the inner rotor and \( \sim 400 \text{ Hz} \) for the outer one, the spectrum collapses to a centreband of linewidth \( \sim 140 \text{ Hz} \) together with double-rotation sidebands (second-order quadrupolar echoes appear in synchronism with cycles of the big rotor) [11]. To denote this particular experimental arrangement, the terms double-rotation \((\text{DOR})\) and second-order spinning \((\text{SOS})\) have been used; we henceforth adopt the former.

It has been pointed out that the effect of double-rotation can also be regarded as an example of rapid rotation around a hinged axis inclined at an angle \( \theta(t) \) with respect to the magnetic field such that

\[
\langle P_l(\cos \theta) \rangle = \int P_l(\cos \theta) W(\theta) \, d\theta = \int_0^T P_l(\cos \theta(t)) \, dt = 0,
\]

for a set of \( l \), an approach dubbed dynamic angle spinning \((\text{DAS})\) [5, 6]. \( W(\theta) \) is a probability distribution for \( \theta \). Equation (5) is obviously solved for all \( l \) by an
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Figure 2. N.M.R. experiments on the central (1/2 ↔ −1/2) transition of sodium-23 in polycrystalline sodium oxalate at 105 MHz. The isotropic shift is at 0 Hz in the figure. Top: static sample, linewidth dominated by dipole-dipole and second-order quadrupolar couplings. Middle: magic-angle spinning (MAS) at ~4000 Hz, displaying a characteristic $I = 4$ lineshape due to averaged second-order quadrupolar coupling. Lower: spectrum under double-rotation (DOR) frequencies of ~2000 Hz and ~400 Hz, illustrating the averaging of second-order line-broadening effects.
isotropic distribution $W(\theta) = (1/2) \sin \theta$. Double-rotation corresponds to
$$\cos \theta(t) = \cos \theta(2) \cos \theta(4) + \sin \theta(2) \sin \theta(4) \cos (\omega_R t + \gamma).$$
Experimental results have indeed been achieved [12] using two orientations of the rotor axis, for example
$$\theta(0) = 37.4^\circ$$
and
$$\theta(T) = 79.2^\circ$$
in both quadrupolar and spin-1/2 systems. Extension of our arguments to other sets of ranks \{l\}, and
therefore to higher-order broadening, is straightforward. Other rotor designs have
also been examined [17]. Finally, it may occasionally prove advantageous to work
in lower magnetic fields thereby enhancing, by means of isotropic second-order (and
perhaps higher-order) shifts, the frequency dispersion normally provided by chemi-
cal shifts alone. Particularly intriguing is the promise of such general sample-
oreorientation trajectories in the context of Tycko's recent scalar reconstruction

techniques [18].

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The centreband linewidth is determined largely by incompletely averaged dipolar couplings and by third-order quadrupolar broadening. The sideband envelope is of course approximated by a sum of spectra from the rapidly rotating small rotor over the orientations of its axis on the cone.

LLOR, A., and Virlet, J. (to be published). Chingas, G. C., Mueller, K. T., Sun, B. Q., Terao, T., and Pines, A. (to be published). There is a continuum of solutions \((\theta_1, \theta_2, k)\) to the equations: 
\[
P_2(\cos \theta_1) = -|k| P_2(\cos \theta_2), \quad P_4(\cos \theta_1) = -|k| P_4(\cos \theta_2).
\]


The axis of a single rotor can be made to precess in (for example) an Andrew-type geometry [1]. Indeed, it has long been known (Schaefer, J. (private communication), Kenwright, A. M., and Harris, R. K. (private communication), VanderHart, D. L., et al. in [16]) that inadvertent 'wobbling' of the spinner axis gives rise to additional sidebands in MAS studies.