NON-ABELIAN GEOMETRIC PHASE FROM INCOMPLETE QUANTUM MEASUREMENTS

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We study the evolution of a quantum system due to a sequence of incomplete measurements, each of which projects the state into an n-dimensional subspace V_n of the Hilbert space of dimension $n+m$. It is shown that the effect of a measurement that projects V_n to V'_n is to parallel transport an orthonormal basis of V_n along the shortest geodesic joining V_n and V'_n in the Grassmann manifold $G_{n,m}$ and transform it by a Hermitian matrix whose eigenvalues are in $(0,1]$. The effect of a dense sequence of measurements leading to a cyclic evolution of V_n is the holonomy transformation associated with the corresponding curve in $G_{n,m}$, with respect to the natural $U(n)$ connection over $G_{n,m}$.

Pancharatnam [1] was perhaps the first to realize that a geometric phase is acquired during a cyclic evolution of the polarization of light due to a sequence of filtering measurements due to a polarizer. Berry [2] showed the existence of the geometric phase [3] in a cyclic evolution governed by the Schrödinger equation with an adiabatically varying Hamiltonian. It was subsequently shown [4] that the restriction of adiabaticity is not necessary and that a state acquires a geometric phase in any cyclic evolution due to the holonomy of a connection [4,5] over the projective Hilbert space $\mathscr{P}$, i.e. the set of rays of a Hilbert space $\mathscr{H}$. The geometric phase was then further generalized to non-linear [6] and non-unitary [7] cyclic evolutions. Also, Pancharatnam's scheme for light [1,8] was generalized to a cyclic evolution in any Hilbert space due to a sequence of measurements [9–11]. Thus it became clear that there is a geometric phase for any cyclic evolution, regardless of how this evolution takes place.

Berry's work inspired the generalization of the adiabatic geometric phase for the cyclic evolution of a degenerate eigensubspace of the Hamiltonian by Wilczek and Zee [12]. This non-Abelian geometric phase was generalized to non-adiabatic evolutions by Anandan [13,7] who described it as due to the holonomy of the connection over the Grassmann manifold, i.e. the set of n-dimensional subspaces of the Hilbert space, where n is the dimension of the subspace undergoing the cyclic evolution. In this paper we generalize it even further by extending it to a cyclic evolution resulting from a sequence of filtering measurements of commuting observables that do not form a complete set. The result of each such measurement is to collapse or project the state into an eigensubspace of this set of observables. We call such a measurement an incomplete measurement as opposed to a complete measurement which is a measurement of a complete set of commuting observables which projects the state into a one-dimensional eigensubspace of this set.

We shall also restrict our treatment to the special class of filtering measurements defined as follows: This is a measurement associated with an eigensubspace of a set of commuting observables with projection operator P such that if the state of the combined system just before the measurement was $|\alpha\rangle|\psi\rangle$, where $|\alpha\rangle$ and $|\psi\rangle$ were the states of the apparatus and the sample (the system to be observed), respectively, then the state just after the measurement is $|\alpha\rangle P|\psi\rangle + |\alpha_1\rangle|\psi_1\rangle$, i.e. the apparatus does not interact with the state $P|\psi\rangle$, but interacts with $(1-P)|\psi\rangle$. So that if the state $|\alpha\rangle P|\psi\rangle$ is subsequently separated from the superposition, it has a well defined phase relation with the initial state $|\alpha\rangle|\psi\rangle$. Since $|\alpha\rangle$ is common to both states we can say that the initial and final states of the *sample* have a well defined phase relation. From now on, we shall consider only the states of the sample. It is also convenient to use the Heisenberg picture in which a state changes only when a measurement is made. We can then disregard the effect of dynamical evolution which gives a dynamical phase in the Schrödinger picture which is treated elsewhere [4,10].

In the complete filtering measurement considered by Pancharatnam [1], the apparatus is the polarizer, $|\psi\rangle$ is the polarization state of the incident light and $P|\psi\rangle$ is the polarization state of the light that passes through the polarizer. Normalizing $|\psi\rangle$ so that $\langle\psi|\psi\rangle=1$, the inner product $(P|\psi\rangle, P|\psi\rangle)=\langle\psi|P|\psi\rangle$ is the ratio of the intensities of the beam passing through the polarizer and the original beam.

Another example of a complete filtering measurement is a Stern–Gerlach experiment in which a beam of spin-$\frac{3}{2}$ particles is split into four by an inhomogeneous magnetic field and one of the beams is selected. This interaction is described by the Hamiltonian $-\hbar\omega(\boldsymbol{r})\boldsymbol{J}\cdot\boldsymbol{n}$, where $\boldsymbol{n}$ is a unit vector fixed with respect to the apparatus and $\boldsymbol{J}$ the angular momentum. Suppose that in this type of experiment the beam interacts instead with an inhomogeneous electric field gradient, described by the degenerate interaction Hamiltonian

$$H=-\hbar\omega(\boldsymbol{r})(\boldsymbol{J}\cdot\boldsymbol{n})^2 .$$

After the measurement the state will be in an eigenstate of $(\boldsymbol{J}\cdot\boldsymbol{n})^2$ whose eigenvalues are $\frac{1}{4}$, $\frac{1}{4}$, $\frac{9}{4}$ and $\frac{9}{4}$. Hence the initial state $|\psi\rangle$ will split into two with each state belonging to an eigenspace spanned by two degenerate eigenstates of $(\boldsymbol{J}\cdot\boldsymbol{n})^2$. Hence the incoming beam will split into two and selecting the "degenerate" beam corresponding to $(\boldsymbol{J}\cdot\boldsymbol{n})^2=\frac{1}{4}$ would be an incomplete filtering measurement. The final state is then $P|\psi\rangle$ multiplied by a dynamical phase factor due to the interaction, where P is the projection operator corresponding to the degenerate subspace.

We shall first review the results for the geometric phase resulting from complete measurements for which P projects an arbitrary state $|\psi\rangle$ to a one-dimensional subspace of $\mathscr{H}$ which is an element of $\mathscr{P}$. This defines a one-to-one correspondence between the set of projection operators of complete measurements and $\mathscr{P}$. We shall therefore denote a point in $\mathscr{P}$ by the corresponding projection operator. If $|\psi'\rangle=\langle\psi|P|\psi\rangle^{-1/2}P|\psi\rangle$ (the normalized projected state) and $|\psi\rangle$ are non-orthogonal then there are two geodesics in $\mathscr{P}$ which join $|\psi\rangle\langle\psi|$ and $|\psi'\rangle\langle\psi'|$, where the geodesics are defined with respect to the Fubini–Study metric [5]. Then it can be shown [9–11] that $|\psi'\rangle$ is obtained by parallel transporting $|\psi\rangle$ along the shortest geodesic joining $|\psi\rangle\langle\psi|$ and $|\psi'\rangle\langle\psi'|$. If we parallel transport $|\psi\rangle$ along the longer geodesic then we will obtain $-|\psi'\rangle$. It follows that the Pancharatnam phase difference χ between any two non-orthogonal states $|\tilde{\psi}\rangle$ and $|\tilde{\psi}'\rangle$ defined by

$$e^{i\chi}=\frac{\langle\tilde{\psi}|\tilde{\psi}\rangle}{|\langle\tilde{\psi}|\tilde{\psi}\rangle|} \tag{1}$$

is 0 or π if $|\tilde{\psi}'\rangle$ is the parallel transport of $|\tilde{\psi}\rangle$ along the shortest or longest geodesic joining $|\tilde{\psi}\rangle\langle\tilde{\psi}|$ and $|\tilde{\psi}\rangle\langle\tilde{\psi}'|$, respectively. Also, if we perform a sequence of measurements resulting in a cyclic evolution, the final state may be obtained by parallel transporting the initial state along the geodesic polygon C formed by the shortest geodesics joining the points on $\mathscr{P}$ corresponding to the projection operators of the successive measurements. Hence, the final and initial states would differ by a phase $\beta=\int_C B=\int_S G$ where S is a surface spanned by C, $B=\langle\tilde{\psi}|\mathrm{d}\tilde{\psi}\rangle$, $|\tilde{\psi}\rangle$ being a differentiable normalized vector field on C and d being the exterior differential on $\mathscr{P}$, and the curvature $G=\mathrm{d}B=\langle\mathrm{d}\tilde{\psi}|\wedge|\mathrm{d}\tilde{\psi}\rangle$.

The fact that G and therefore β are invariant under the gauge transformation $|\tilde{\psi}\rangle\to e^{ig}|\tilde{\psi}\rangle$, where g is a real differentiable function on $\mathscr{P}$, suggests that it should be possible to express them entirely in terms of $\mathscr{P}$, i.e. the set of projection operators of the form

$P=|\psi\rangle\langle\psi|$. Indeed, it can be shown that the value of G at $P\in\mathscr{P}$ is

$$G(P)=\mathrm{tr}(P\,\mathrm{d}P\wedge\mathrm{d}P)=\langle\psi|\mathrm{d}P\wedge\mathrm{d}P|\psi\rangle\ . \qquad (2)$$

Hence the operator $\hat{G}=\mathrm{d}P\wedge\mathrm{d}P$ is an elegant expression for the curvature entirely in terms of the gauge invariant P. It is, however, not possible to express B in terms of P alone because B depends on the choice of gauge.

Consider now the cyclic evolution of an n-dimensional subspace of the $(n+m)$-dimensional $\mathscr{H}$. Let $\mathrm{G}_{n,m}$ be the Grassmann manifold consisting of all the n-dimensional subspaces of $\mathscr{H}$. We call a set of n orthonormal vectors $\{|\psi_i\rangle,\ i=1,2,\ldots,n\}$ an n-frame. An n-frame spans a subspace $\mathrm{V}_n\in\mathrm{G}_{n,m}$ with the associated projection operator

$$P=\sum_{i=1}^{n}|\psi_i\rangle\langle\psi_i|\ .$$

Physically, $n^{-1}P$ may also be regarded as a thermal density matrix. Clearly, P is independent of the chosen orthonormal basis $\{|\psi_i\rangle\}$ of V_n and is therefore invariant under the unitary group $\mathrm{U}(n)$ of transformations between the orthonormal bases of V_n. Therefore $\mathrm{G}_{n,m}$ can be identified with the set of projection operators P uniquely associated with the subspaces V_n. Also, the set of bases of $\mathscr{H}$, or $(n+m)$-frames, can be identified with the group $\mathrm{U}(n+m)$, and V_n with the equivalence class of $(n+m)$-frames each consisting of n vectors in V_n and m vectors in the orthogonal complement V_m of V_n in $\mathscr{H}$. Therefore we can also make the identification $\mathrm{G}_{n,m}=\mathrm{U}(n+m)/\mathrm{U}(n)\times\mathrm{U}(m)$. Also, the Stiefel manifold $\mathrm{S}_{n,m}$, defined to be the set of n-frames, has the identification $\mathrm{S}_{n,m}=\mathrm{U}(n+m)/\mathrm{U}(m)$. It follows that we may also write $\mathrm{G}_{n,m}=\mathrm{SU}(n+m)/\mathrm{S}(\mathrm{U}(n)\times\mathrm{U}(m))$ and $\mathrm{S}_{n,m}=\mathrm{SU}(n+m)/\mathrm{SU}(m)$.

We therefore have the following tower of bundles: $\mathrm{U}(n+m)$ is a $\mathrm{U}(m)$-principal fiber bundle over $\mathrm{S}_{n,m}$ with projection map Φ (say), while $\mathrm{S}_{n,m}$ is a $\mathrm{U}(n)$-principal fiber bundle over $\mathrm{G}_{n,m}$ with projection map Π. Also, $\mathrm{U}(n+m)$ may be regarded as a principal fiber bundle over $\mathrm{G}_{n,m}$ with $\mathrm{U}(n)\times\mathrm{U}(m)$ as the structure group and projection map $\chi=\Pi\Phi$. There is a natural connection in the bundle $\mathrm{S}_{n,m}$ over $\mathrm{G}_{n,m}$ whose connection one-form with respect to a field of n-frames $\{|\tilde{\psi}_i\rangle\}$ on $\mathrm{G}_{n,m}$ is $B_{ij}=\mathrm{i}\langle\tilde{\psi}_i|\mathrm{d}\tilde{\psi}_j\rangle$. The orthonormality of $\{|\tilde{\psi}_i\rangle\}$ implies that B_{ij} is a Hermitian matrix, i.e. it is in the Lie algebra of $\mathrm{U}(n)$. It was shown [7,12,13] that this connection gives the non-Abelian geometric phase in the cyclic evolution of a subspace that is given by a closed curve C in $\mathrm{G}_{n,m}$.

The curvature or Yang–Mills field of this connection with respect to the given n-frame field on $\mathrm{G}_{n,m}$ or gauge is $G_{ij}=\mathrm{d}B_{ij}+B_{ik}\wedge B_{kj}$, using the summation convention. Under a unitary gauge transformation $|\tilde{\psi}_i\rangle\to U_n|\tilde{\psi}_i\rangle$, $B\to U_n^+BU_n+\mathrm{i}U_n^+\mathrm{d}U_n$ and $G\to U_n^+GU_n$. Also, a straightforward computation shows that

$$G_{ij}(P)=\langle\psi_i|\mathrm{d}P\wedge\mathrm{d}P|\psi_j\rangle\ , \qquad (3)$$

where there is a wedge product as well as operator multiplication between the two dP's, which generalizes (2). Hence, again, we have an elegant curvature operator $\hat{G}=\mathrm{d}P\wedge\mathrm{d}P$ in this non-Abelian case. $G_{ij}(P)$ are the coefficients of $\hat{G}$ in the arbitrary basis $\{|\psi_i\rangle\}$ of the subspace V_n corresponding to the projection operator P.

We shall now define metrics on the above three bundles as follows: A tangent vector X of $\mathrm{U}(n+m)$ may be represented by an $(n+m)\times(n+m)$ Hermitian matrix. A metric h is defined by the condition $h(X,Y)=\mathrm{tr}(XY)$, where X and Y are tangent vectors at any point in $\mathrm{U}(n+m)$. This metric is real and positive definite. When restricted to the $\mathrm{SU}(n+m)$ subgroup of $\mathrm{U}(n+m)$, h is the Cartan–Killing metric up to a constant factor. Let g be the metric in $\mathrm{S}_{n,m}$ such that Φ is a Riemannian submersion, i.e. dΦ is an isometry when restricted to the orthogonal complement of the kernel of dΦ. Also, let f be a metric in $G_{n,m}$ such that Π is a Riemannian submersion. In the special case of $n=1$, $G_{1,m}$ is the m-dimensional complex projective space and f is the Fubini–Study metric. The horizontal spaces of the connection defined above in the bundle $\mathrm{S}_{n,m}$ are orthogonal to the fibers of $\mathrm{S}_{n,m}$ with respect to the metric g, which may be regarded as an alternative definition of this connection. But we also have natural connections in $\mathrm{U}(n+m)$ regarded as bundles over $\mathrm{S}_{n,m}$ or $\mathrm{G}_{n,m}$ defined by horizontal spaces that are orthogonal to the corresponding fibers with respect to the metric h.

Consider now two successive incomplete measurements into subspaces V_n and V'_n with associated projection operators P and P'. Let $\{|\tilde{\psi}_i\rangle\}$ and $\{|\tilde{\psi}'_j\rangle\}$ be two arbitrary orthonormal bases of V_n and V'_n, respectively. Then $\tilde{Z}_{ij}=\langle\tilde{\psi}'_i|P'P|\tilde{\psi}_j\rangle$ is an $n\times n$

matrix which maps V_n into V'_n. We shall say that V_n and V'_n are anti-orthogonal if the matrix Z is non-singular, i.e. no vector in V_n is orthogonal to every vector of V'_n or vice versa. We now make the polar decomposition of $\tilde{Z}$ according to $\tilde{Z}=\tilde{N}\tilde{U}$, where $\tilde{N}$ is a non-negative matrix (i.e. a Hermitian matrix with non-negative eigenvalues) and $\tilde{U}$ is a unitary matrix. Here, $\tilde{N}=(\tilde{Z}^+\tilde{Z})^{1/2}$ is unique, and $\tilde{U}$ is unique if and only if $\tilde{Z}$ is non-singular. Hence, if V_n and V'_n are anti-orthogonal, as we shall assume from now onwards, the uniquely defined $\tilde{U}$ is a generalization of the Pancharatnam phase factor (1).

Suppose we now change the bases according to $|\tilde{\psi}_i\rangle = U_{1ij}|\psi_j\rangle$ and $|\tilde{\psi}'_i\rangle = U_{2ij}|\psi_j\rangle$. Then $\tilde{Z}$ transforms to $Z=U_2\tilde{Z}U_1^+=NU$, where $N=U_2\tilde{N}U_2^+$ is non-negative and $U=U_2\tilde{U}U_1^+$ is unitary. For a given basis in V_n ($U_1=I$), there is a unique basis in V'_n ($U_2=\tilde{U}^+$) for which $U=I$. Similarly, for a given basis in V'_n ($U_2=I$), there is a unique basis in V_n ($U_1=\tilde{U}$) for which $U=I$. Given such a pair of bases, for which our non-Abelian generalization of the Pancharatnam phase is the identity, we shall say that the two bases are parallel. This defines a distant parallelism on $G_{n,m}$. On choosing U_2 so that N is diagonal and $U_1=U_2\tilde{U}$, the corresponding parallel bases are such that $Z=N=\text{diag}(\cos\theta_1, \cos\theta_2, ..., \cos\theta_n)$, $\theta_i\in[0, \pi/2]$. This is like the Abelian case in that each $|\psi_i\rangle = P|\psi_i\rangle$ is projected to $P'|\psi_i\rangle = \cos\theta_i\,|\psi'_i\rangle$ during the second measurement. But the non-Abelian nature of incomplete measurements manifests itself when we have a cyclic evolution of a subspace. If we then choose the bases in the subspaces defined by the incomplete measurements such that Z for any two successive subspaces is non-negative, the initial and final bases which project to the same $P\in G_{n,m}$ are related by a non-trivial unitary transformation, in general.

We now show that two parallel bases in subspaces corresponding to P, $P'\in G_{n,m}$ are related by parallel transport along the shortest geodesic C joining P and P', the geodesic being defined with respect to the metric f on $G_{n,m}$. To prove this it is convenient to horizontally lift C to a curve Γ in $U(n+m)$ regarded as a principal fiber bundle over $G_{n,m}$. Then Γ is a geodesic with respect to the metric h and it projects to a horizontal geodesic γ in $S_{n,m}$ regarded as a principal fiber bundle over $G_{n,m}$. Also, γ projects to C. Now choose an orthonormal basis of $\mathscr{H}$ to consist of n vectors in the subspace V_n corresponding to P and m vectors in the orthogonal complement V_m of V_n. This basis may be regarded as a point in the fiber in $U(n+m)$ over P, which we take to be an end point of Γ. In this basis, $\Gamma(t)$ is the curve $\exp(itH)$ where the Hermitian matrix H has the form

$$H=\begin{pmatrix} 0 & A \\ A^+ & 0 \end{pmatrix},$$

where A is an $n\times m$ matrix. This is because the action along the fibers is generated by matrices of the form

$$J=\begin{pmatrix} B & 0 \\ 0 & C \end{pmatrix},$$

where B and C are arbitrary Hermitian matrices of order n and m respectively, and we must have $h(J, H)=\text{tr}(JH)=0$, since $\Gamma(t)$ is horizontal.

Since $\gamma(t)$ is horizontal if we change the chosen n-frame in V_n and the m-frame in V_m then H transforms to WHW^+, where

$$W=\begin{pmatrix} U_{(n)} & 0 \\ 0 & U_{(m)} \end{pmatrix},$$

$U_{(n)}$ being an $n\times n$ unitary matrix. Therefore, A transforms to $U_{(n)}AU_{(m)}^+$. This change of bases moves $\Gamma(t)$ and $\gamma(t)$ vertically in the bundles $U(n+m)$ and $S_{n,m}$ respectively, but does not change C. Clearly, these bases can be chosen so that the transformed A has a real number x_i as the (i, i)th coefficient, $1\leqslant i\leqslant \min\{m, n\}$, and all other coefficients are zero. Then the ith column of $\exp(itH)$, for $1\leqslant i\leqslant n$, has $\cos\theta_i$ as the ith element and $i\sin\theta_i$ as the $(n+i)$th element with all other elements zero, where $\theta_i=tx_i$. Our matrix

$$Z_{ij}=\langle\psi'_i|\psi_j\rangle=\langle\psi_i|\exp(-itH)|\psi_j\rangle$$
$$=\text{diag}(\cos\theta_1, ..., \cos\theta_n)\,.$$

For the shortest geodesic joining the two extremities of C, $0\leqslant\theta_i\leqslant\pi/2$ for all i from 0 to n. Therefore Z is non-negative in this case. If the basis in V_n is changed then Z undergoes a unitarity similarity transformation and so remains non-negative.

Now the basis states $|\psi'_i\rangle=\exp(itH)|\psi_i\rangle$, $i=1, 2, ..., n$ are obtained from $|\psi_i\rangle$ by parallel transporting along the shortest geodesic C using the above connection in the bundle $S_{n,m}$. Since Z is then positive we have shown that distant parallelism defined above is the same as parallel transporting along the shortest geodesic. Also, since in the preferred basis in which Z is diagonal, each $|\psi_i\rangle$ will be projected to $\cos\theta_i\,|\psi_i\rangle$ when the measurement is made. Therefore $\cos^2\theta_i$ physically represents the reduction of intensity of a beam, initially in the state $|\psi_i\rangle$, during the measurement. Since the projection operator is linear, a linear combination $\sum_{i=1}^{n} a_i\,|\psi_i\rangle$ goes over to $\sum_{i=1}^{n} a_i \cos\theta_i\,|\psi_i\rangle$.

Hence if we have a sequence of projections that give a cyclic evolution of an n-dimensional subspace, then the distant parallelism introduced here enables us to construct a sequence of n-frames from a given initial n-frame. The final n-frame will then be related to the initial frame, which is also a basis of the initial subspace V_n by a unitary transformation. It follows from the theorem proved above that this transformation is the holonomy transformation due to parallel transport around the geodesic polygon formed by joining successive points in $G_{n,m}$, defined by the measurements, by the shortest geodesics. Also, the result of this sequence of measurements on an arbitrary state in V_n, with respect to any orthonormal basis of V_n, is to multiply by a sequence of positive matrices and the unitary matrix of the above holonomy transformation. But the determination of the positive matrices requires knowledge of each projection. This is unlike the Abelian case [9,10] where the acquired phase is determined entirely by the holonomy transformation.

However, if a dense sequence of measurements are made on the system so that the final subspace is the same as the initial subspace V_n, then the effect on an arbitrary state of V_n is just the non-Abelian holonomy transformation mentioned above. This is because, in the above analysis, $\cos\theta_i$ is 1 if $O(\theta_i^2)$ is neglected and therefore the positive matrices tend to the unit matrix in the limit of dense measurements. Thus in this limiting case the effect of the measurements is a unitary transformation which is purely geometrical.

Recent magnetic resonance experiments on quadrupolar systems that observe Abelian and non-Abelian holonomy [14] can be extended to the incomplete measurements studied in this Letter and the predicted results can thus be experimentally verified.

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