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Relaxation-selective magnetic resonance imaging

Seth D. Bush [a], David E. Rourke [b,*], Lana G. Kaiser [a], Alexander Pines [a]

[a] Materials Sciences Division, Lawrence Berkeley National Laboratory and Department of Chemistry, University of California, Berkeley, CA 94720, USA

[b] Magnetic Resonance Centre, School of Physics and Astronomy, Nottingham University, Nottingham NG7 2RD, UK

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Abstract

The Bloch equations with T_2 relaxation can be inverted in closed form with respect to T_2, using inverse scattering theory. Hence, radio frequency pulses can be calculated that cause a final magnetization response that is any desired function of T_2, provided that function is physically realizable (however, there are strong constraints on what is physically realizable). A useful subclass of such pulses are 'dressing' pulses, which store the magnetization on the z-axis, with magnitude a given function of T_2. This enables spins to be selectively nulled according to their T_2 – this is demonstrated by obtaining a relaxation-selective image of a phantom.

1. Introduction

When radio frequency (rf) pulses are applied to a system made up of spins that are distinguished by one or more parameters (such as free-precession frequency, or T_2 relaxation time), the final state of the spins will be a function of those parameters. The rf pulse design problem involves finding that rf pulse which leaves the spin system in a given desired final state, or as close to it as possible.

This problem is most commonly approached in the 'forward' direction. For example, rf pulses that give a desired magnetization response as a function of resonance offset ('frequency-selective pulses') are often designed by numerically searching through a space of allowed pulses, calculating the magnetization response for each trial pulse. Typical algorithms used are gradient-based minimization [1–3] and simulated annealing [4–6].

For sufficiently restricted spaces of allowed pulses, the determination of the best pulse shape can be done more simply. For example, rf pulses that give a desired magnetization response as a function of T_2 relaxation time ('T_2-selective pulses') have been obtained by calculating the response of a rectangular soft pulse [7]. This response can be given in closed form – it is then easy to find rectangular pulses that suppress spins with either long or short T_2 times. In a similar manner, T_1-contrast enhancement has been obtained by calculating the response of a train of hard pulses [8].

This Letter is concerned with the calculation of T_2-selective pulses via an *inverse* method. Firstly, constraints on the total space of allowed magnetization responses, as functions of T_2, are obtained. It is

* Corresponding author. E-mail: rourke@magres.nottingham.ac.uk

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important to do this, as the space is quite restrictive. For example, it is not possible for truly 'notch filter' pulses to exist, as they do with frequency-selective pulses.

Secondly, inverse scattering theory allows the exact calculation of the rf pulse that will give any allowable response [9,10]. This Letter concentrates on an important subset of allowable responses. It is important because the corresponding pulses are able to selectively null spins with particular T_2 values, and also because the pulses can be calculated more simply than in the general case, using the 'dressing method' – a tool from inverse scattering theory [10]. Such pulses are therefore 'notch filter' pulses, to within the constraints described below. This complements the use of a 90° pulse followed by a wait period, which would constitute a high pass T_2 filter, and the square pulses described in Ref. [7], which were used as low pass T_2 filters.

A pulse was calculated using this method to null magnetization of spins with a particular T_2 value, leaving other spins as unaffected as possible. This same pulse (after scaling its duration and amplitude) could be used to null spins with different T_2 values to that originally specified. It was tested experimentally as part of imaging a two-component system, consisting of a short T_2 spin species, physically separated from a long T_2 species. By nulling first the short T_2 species (leaving the long T_2 species alone), and then nulling the long T_2 species (leaving the short T_2 species alone), selective images of the different T_2 species were obtained.

2. Theory

Inverse scattering theory allows the 'inversion' of systems of the form

$$\frac{\partial \Phi}{\partial t} = [\mathrm{i}\xi J + V(t)]\Phi(\xi,t), \qquad (1)$$

where $\Phi(\xi,t)$, J, and $V(t)$ are $n \times n$ matrices, and ξ is a scalar (the scattering parameter). It is possible, and often convenient, to make all quantities in the above equation dimensionless (e.g., the time t is expressed as a multiple of some convenient characteristic time). This has been done throughout this section.

Given an initial specification of Φ as a function of ξ, together with a 'physically realizable' final $\Phi(\xi)$, inverse scattering theory allows the determination of $V(t)$ [11]. The Bloch equations with T_2 relaxation can be written in the form of Eq. (1), with $n = 3$ [10]. In this case, ξ corresponds to the T_2 relaxation rate, $\Gamma_2 = 1/T_2$. Φ corresponds to the magnetization vector, and V to the rf pulse.

Due to the particular form of the Bloch equations, there are constraints on the allowable final magnetization, (m_x, m_y, m_z), as a function of Γ_2 after an rf pulse. The most important constraints are that the final longitudinal magnetization, m_z, is analytic in the right half complex Γ_2 plane [9,10], and that $m_z \to 1$ as $|\Gamma_2| \to \infty$. It is also bounded in magnitude, $|m_z| \leqslant 1$, when Γ_2 is imaginary. (The previous two statements assume that the magnetization is normalized so its magnitude equals 1 at equilibrium.)

Standard results from complex analysis can be used to obtain some physically obvious results on the bounds of m_z for $\Gamma_2 \in \bar{\mathfrak{R}}^+$, where $\bar{\mathfrak{R}}^+$ is the closed positive real axis, $0 \leqslant \Gamma_2 \leqslant \infty$. Apply the conformal map $\Gamma_2' = (\Gamma_2 - 1)/(\Gamma_2 + 1)$, to map the right half complex Γ_2 plane to the unit disk, $|\Gamma_2'| \leqslant 1$, with the imaginary axis mapped to the disk's boundary. Then, from above, $|m_z| \leqslant 1$ on the boundary, and m_z is analytic inside the disk. Hence, the maximum modulus theorem [12] shows that either $|m_z| < 1$ everywhere inside the disk, or that $|m_z| = 1$ everywhere (including the boundary). Hence $|m_z| \leqslant 1$ for $\Gamma_2 \in \bar{\mathfrak{R}}^+$, with the proviso that if $|m_z| = 1$ for *any* Γ_2 in the open interval $\mathfrak{R}^+$ (i.e., $0 < \Gamma_2 < \infty$), then $|m_z| = 1$ for *all* $\Gamma_2 \in \bar{\mathfrak{R}}^+$. This is basically proving that the magnetization vector cannot ever increase in length above its equilibrium value, and that its length will be less than its equilibrium value once it has been moved off the z axis, unless it has no T_2 relaxation (or infinitely fast T_2 relaxation – in which case it can never move off the z axis).

Suppose it is desired to construct an rf pulse that will cause $m_z = 0$ for some value, say γ_2, of $\Gamma_2 \in \mathfrak{R}^+$. Such a pulse would enable the nulling of the magnetization of spins with $T_2 = 1/\gamma_2$. Without loss of generality, γ_2 can be taken equal to 1. Then the function $m_z(\Gamma_2)$ is mapped by the above conformal transformation so that m_z is zero at $\Gamma_2' = 0$. Then the Schwarz lemma [12] implies that $|m_z(\Gamma_2')| \leqslant |\Gamma_2'|$ within the unit disk. Hence if $m_z = 0$ at $\Gamma_2 = 1$, then

$|m_z| \leqslant |(\Gamma_2 - 1)/(\Gamma_2 + 1)|$ for all Γ_2 in the right half complex plane. In general, if m_z is required to equal 0 for $\Gamma_2 = \gamma_2$, with γ_2 not necessarily equal to 1, then m_z must obey the constraint

$$|m_z| \leqslant \left| \frac{\Gamma_2 - \gamma_2}{\Gamma_2 + \gamma_2} \right| \tag{2}$$

for Γ_2 in the right half complex plane.

This result means that rf pulses cannot be used to obtain arbitrarily sharp notch filter m_z responses as functions of Γ_2. For example, consider the notch filter function:

$$m_z(\Gamma_2) = \begin{cases} 1 & \text{for } \Gamma_2 < 0.5\,, \\ 0 & \text{for } 0.5 < \Gamma_2 < 1.5\,, \\ 1 & \text{for } \Gamma_2 > 1.5\,. \end{cases} \tag{3}$$

It would not be possible to obtain a response that were arbitrarily close to this desired response. If a pulse were designed that gave $m_z(\Gamma_2 = 1) = 0$, then the maximum achievable value of $m_z(\Gamma_2 = 1.5)$ is $0.5/2.5 = 0.2$. This is in contrast to pulses used for frequency selection, where a response arbitrarily close to the above (replacing Γ_2 by frequency offset) could be obtained.

An important class of final responses has, given initial magnetization (0,0,1), a final response

$$m_x = m_y = 0\,, \quad \text{and}$$

$$m_z = \prod_{j=1}^{r} \frac{\Gamma_2 - g_j}{\Gamma_2 + g_j^\star}\,. \tag{4}$$

Here, * means the complex conjugate. Parameters g_j may be chosen arbitrarily, as can the number of parameters, r, except that all g_j must be in the right half complex plane, and any g_j off the real axis must be in a $(g_j, g_j^\star)$ pair.

A pulse giving such a response would store magnetization on the z axis, with magnitude dependent on T_2. The pulse could be chosen to null spins with any desired T_2 value (more than one T_2 could be nulled at the same time). For each g_j on the real axis, the magnetization will be nulled at $T_2 = 1/g_j$.

One reason for the importance of these pulses is that they can be easily calculated from the desired response with a tool from inverse scattering theory – the dressing method [10,13]. This is in contrast to the inversion of a general response, which requires the solution of coupled linear integral equations [9] (unless the corresponding pulse is required to be real, in which case simpler inverse scattering theory for a second-order system can be used [10]). A pulse with the response of Eq. (4) will be called an rth-order dressing pulse. Note that even-order dressing pulses behave differently to odd-order pulses at $\Gamma_2 = 0$. For even-order pulses, $m_z = +1$. For odd-order pulses, $m_z = -1$. For both, $m_z \to 1$ as $\Gamma_2 \to \infty$.

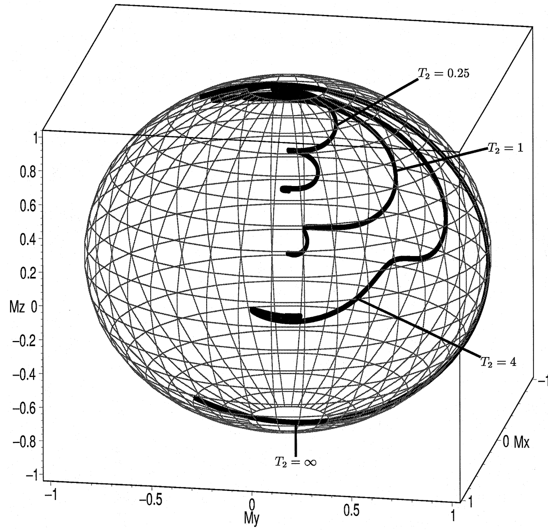

Fig. 1. The evolution of the magnetization, for a series of spins with different T_2 values, during a third-order dressing pulse designed to null species with a T_2 of 1. All magnetizations start at (0,0,1) and move to a position on the z axis dependent on their T_2. (The path of the $T_2 = \infty$ species is not clear; it overshoots the z axis, but then retraces its path to the axis.)

Fig. 1 gives a geometric picture of how a third-order dressing pulse designed to null a spin species with $T_2 = 1$ works. The magnetization with that T_2 is moved from (0,0,1) to (0,0,0). Other magnetizations are stored elsewhere on the z axis, and could then be observed with application of a 90° pulse.

3. Examples

The simplest dressing pulse [10] has the form, in units of angular frequency, $\omega(t) = g\,\mathrm{sech}(gt)$, corresponding to a final magnetization response

$$m_x = m_y = 0\,, \quad m_z = \frac{\Gamma_2 - g}{\Gamma_2 + g}\,. \tag{5}$$

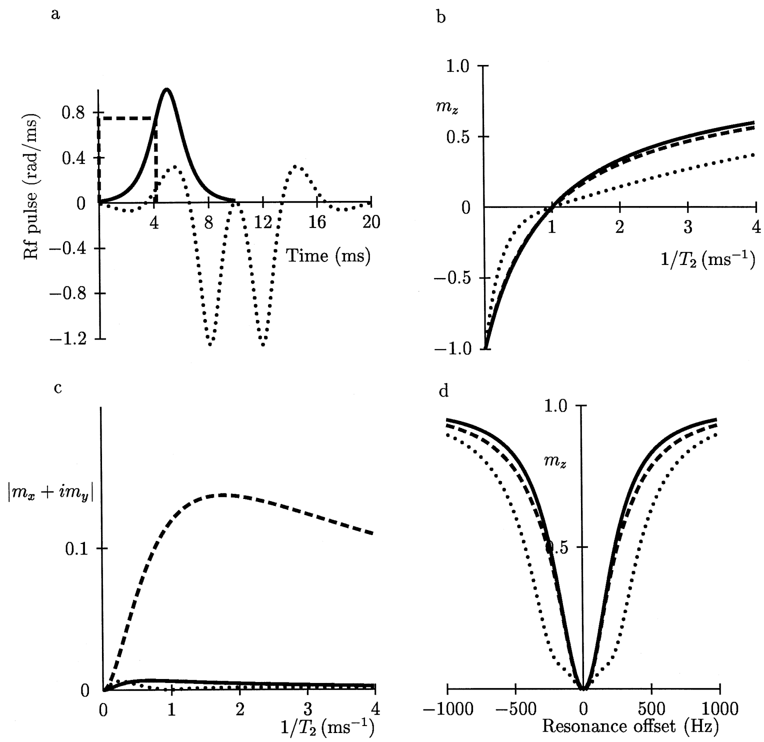

Fig. 2. (a) Pulse shapes for a square pulse, a sech pulse, which is the first-order dressing pulse, and a third-order dressing pulse. (b) and (c) show the longitudinal and transverse magnetization responses versus $1/T_2$ for spins on resonance, obtained by numerical integration of the Bloch equations for the given pulse shapes. The transverse responses of the dressing pulses are close to zero. They are not identically zero as the pulses have been truncated. (d) shows the m_z responses versus resonance offset for spins with $T_2 = 1$ ms after using the three pulse shapes. In all graphs, the dashed, solid, and dotted lines refer to the square, sech, and third-order dressing pulses, respectively.

This pulse therefore nulls spins with $T_2 = 1/g$. Furthermore, by comparison with Eq. (2), it is the pulse that allows maximum contrast between these spins and spins with a different T_2 time.

One problem with this pulse, however, is that it is quite sensitive to resonance offset (when pulses are calculated with inverse scattering theory to have a specified T_2 response, it is assumed that all spins are on resonance). It is possible, just as with the square pulses of Ref. [7], to use hard 180° pulses within the pulse to decrease this sensitivity. However, we found, by numerical simulation, that the third-order dressing pulse with m_z response

$$m_z = \frac{(\Gamma_2 - 1)(\Gamma_2 - [1+i\sqrt{3}]/2)(\Gamma_2 - [1-i\sqrt{3}]/2)}{(\Gamma_2 + 1)(\Gamma_2 + [1+i\sqrt{3}]/2)(\Gamma_2 + [1-i\sqrt{3}]/2)} = \frac{(\Gamma_2 - 1)[(\Gamma_2)^2 - \Gamma_2 + 1]}{(\Gamma_2 + 1)[(\Gamma_2)^2 + \Gamma_2 + 1]} \quad (6)$$

is less resonance-offset sensitive than the sech pulse, and might therefore be a better choice as a relaxation-selective pulse in many cases. See also Fig. 2, which compares the sech pulse, the third-order dressing pulse, and a square pulse designed to null spins with $T_2 = 1$ ms. The third-order dressing pulse corresponds to that shown in Section IVC of Ref. [10].

The figure shows the pulse shapes, the longitudinal and transverse magnetization responses as functions of Γ_2 (for spins on resonance), and the m_z responses as functions of resonance offset (for spins with $T_2 = 1$ ms). All responses were obtained via numerical integration of the Bloch equations for pulse durations as shown in the figure.

Note that the transverse responses for the two dressing pulses are very close to zero for all T_2 rates, whereas that for the square pulse is appreciable (in principle, the transverse responses for the dressing pulses should be zero – their being non-zero is a consequence of the pulses being truncated). Hence, dressing pulses do truly null the magnetization at specified T_2 values, whereas other pulses null just the longitudinal response.

It is interesting that the m_z response of the square pulse is very similar to the response of the sech pulse, although $|m_z|$ for the latter is always greater than or equal to that of the square pulse (as follows from the previous section). The resonance offset behaviour of the third-order dressing pulse is approximately the same as would be obtained from the square pulse shown if two 180° hard pulses were added to the latter at times $T/3$ and $2T/3$, where T is the square pulse's duration.

As a simple test of using a dressing pulse to selectively null spins with a specified T_2, a phantom as shown in Fig. 3a was constructed. This consisted of two concentric water samples, doped with $MnCl_2$, so that the separate components had different T_2 times, but similar resonance offsets ($MnCl_2$ was used as a doping agent, as it has very little effect on the resonance offset of the water). Combined with the homogeneity of the field (10 Hz broadening), all spins in both compartments lay within the range of resonance offsets over which the third-order dressing pulse works.

The two components had different T_1 times. The outer component's T_1 was sufficiently long to be taken as infinite. The inner component had a T_1 of 12 ms. Although the dressing pulses are designed assuming no T_1 relaxation, it can be shown [10] that odd-order dressing pulses work with T_1 present. Finite T_1 has the effect of decreasing the T_2 nulled by the pulse (to what value it is decreased to must be determined by numerical simulation).

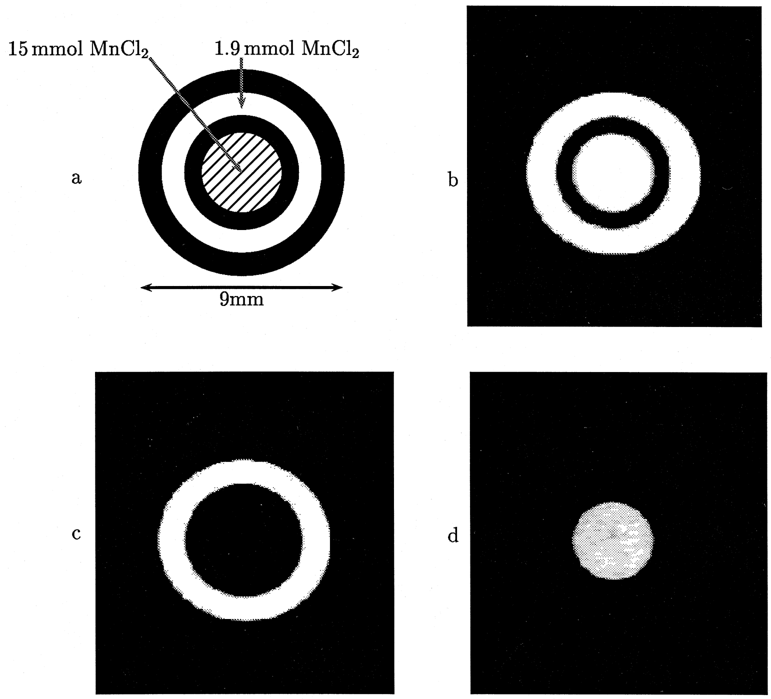

Fig. 3. (a) The doped water phantom used to demonstrate the T_2 selectivity of the third-order dressing pulse. It consisted of two concentric tubes, filled with $MnCl_2$ with concentrations shown. The T_2 times of the outer and inner solutions were 3.79 and 0.45 ms, respectively. The T_1 relaxation of the outer solution was long enough to be taken as infinite, the T_1 time of the inner solution was 12 ms. (b), (c), and (d) show three images obtained from the phantom. In (b), all the spins were excited with a 90° hard pulse prior to imaging. In (c), the fast-relaxing spins were selectively nulled prior to the excitation pulse. In (d), the slow-relaxing spins were selectively nulled prior to the excitation pulse. All images have their intensities to the same scale.

Fig. 3b shows an initial image of both components of the phantom, obtained at 4.2 T using a hard 90° pulse, followed by an imaging sequence.

Subsequent images were obtained by preceding the above sequence with the third-order dressing pulse described above, and shown in Fig. 2a. By scaling the pulse in duration and amplitude, it could be used to selectively null either the fast or the slow relaxing species (e.g., doubling the pulse duration and halving its amplitude will double the T_2 nulled by the pulse). These images are shown in Fig. 3c,d, confirming the ability of this pulse to act as a notch T_2 filter.

4. Conclusions

Relaxation-selective pulses are a useful complement to frequency-selective pulses. They have previously been used to create contrast between spin species with different relaxation times [14], and to selectively null magnetization [7]. They have also been used in magnetization transfer experiments [15].

In contrast to previous work, we have found strong constraints on the physically allowable T_2 responses, and determined how to invert any such response. We have concentrated on 'dressing' pulses as they are relatively easy to calculate, and they work well in nulling the magnetization of one or more spin species distinguished by T_2 – although the constraints on the allowable responses mean that pulses that null multiple T_2 values will work well only if the values are well separated.

For example, we have demonstrated the selective nulling of spins in a two-component system, where the T_2 times are different by about a factor of 8. We also chose the system so that all spins were reasonably close in resonance offset. For a system with a wide range of resonance offsets, a bandwidth-broadening scheme would need to be considered (the scheme used in Ref. [7] works for dressing pulses). However, it is not possible, even in principle, to sharpen the discriminatory ability of rf pulses between species with different T_2 times above a certain limit.

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