Theory of Broadband Heteronuclear Decoupling in Multispin Systems

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The Waugh theory of broadband decoupling in NMR for I–S spin-$\frac{1}{2}$ pairs is extended to arbitrary spin systems. It is shown that complete decoupling is achieved over a certain bandwidth if the irradiation scheme generates an average Hamiltonian for the I spins whose eigenvectors and eigenvalues are independent of resonance offset. If the observed S spins are only weakly coupled, it is possible to calculate the resulting spectrum directly from the offset dependence of the average Hamiltonian of the isolated I-spin system under the influence of the periodic decoupling sequence. The treatment applies to indirectly observed multiple-quantum transitions as well as to directly observable single-quantum resonance lines. © 1987 Academic Press, Inc.

INTRODUCTION

Broadband heteronuclear decoupling in isotropic liquids changed dramatically when cw and noise decoupling were replaced by sequences of phase-shifted radiofrequency pulses (*1, 2*). The improvements were based on sequences of composite pulses that are compensated for off-resonance effects. This approach was put on a sound theoretical basis by Waugh (*3*) who calculated the NMR spectrum of a spin $S = \frac{1}{2}$ coupled to a spin $I = \frac{1}{2}$ which is perturbed by a periodic sequence of radiofrequency pulses. The main result of Waugh's theory is that the decoupling efficiency of a pulse sequence can be analyzed by considering its effect on the irradiated I spin without taking the S spin explicitly into account. If the average rotation frequency Ω of the I spin under the influence of the pulse cycle is calculated as a function of offset, the residual coupling $J_{res}(\delta)$ at an offset δ is given as

$$J_{res}(\delta) = J \frac{\partial \Omega(\delta)}{\partial \delta}, \tag{1}$$

where J represents the unperturbed heteronuclear coupling constant. This result is derived for a single I–S spin pair and an irradiation sequence which is periodic with the same period as the sampling interval for the S-spin signal. The search for a decoupling sequence that works for spins $I > \frac{1}{2}$ in the presence of electric quadrupole interactions as well as for systems of dipole–dipole coupled I spins (*4*) made it necessary to develop a more general theory of heteronuclear decoupling which can be applied to these spin systems. The results of Waugh's theory are not directly applicable since

the effect of a pulse sequence on a spin $> \frac{1}{2}$ or on a system of coupled spins cannot be described in terms of rotations in three-dimensional space. Further it is necessary to take into account the additional interactions that do not occur for individual spins $\frac{1}{2}$. While the presence of electric quadrupole or dipole–dipole interactions does not make it impossible to decouple, even if they exceed the strength of the decoupling field, it makes the decoupling performance much more susceptible to resonance offsets. In the case of spins $I = 1$ it was shown that decoupling can be achieved in the presence of an electric quadrupole interaction that is considerably bigger than the decoupling field, if the frequency of the decoupler is set on resonance with the double-quantum transition (5, 6). However, the effective field strength acting on the double-quantum transition is scaled down by the ratio of the nominal rf field strength divided by the strength of the quadrupole coupling. Accordingly, the decoupling performance is more sensitive to resonance offset by the same ratio. Also the composite pulse decoupling schemes used in isotropic liquids fail in the presence of second-rank interactions. This is not surprising, since the composite pulse schemes were based on the compensation of off-resonance effects by combining appropriate rotation elements and did not include the effect of second-rank interactions.

The organization of the paper is as follows: In the second section we develop the theory for the observation of individual spins $S = \frac{1}{2}$ coupled to an arbitrary system of I spins. The third section extends the theory to systems of weakly coupled S spins of arbitrary magnitude and the final section summarizes the results.

SINGLE SPIN $S = \frac{1}{2}$, COUPLED TO AN ARBITRARY SYSTEM OF I SPINS

Throughout this paper we will denote the spin species that is observed as the S spins and the species that is irradiated as the I spins and assume that $I \neq S$. The Hamiltonian of the spin system can be written in the form

$$\mathscr{H}(t) = \mathscr{H}_S + \mathscr{H}_I + \mathscr{H}_{IS} + \mathscr{H}_{II} + \mathscr{H}_{Irf}(t). \qquad [2]$$

$\mathscr{H}_S$ and $\mathscr{H}_I$ represent single-spin interactions with the magnetic field or the electric field gradient tensor. $\mathscr{H}_{IS}$ and $\mathscr{H}_{II}$ contain spin–spin coupling terms and $\mathscr{H}_{Irf}(t)$ the interaction of the I spins with the radiofrequency field. In high-magnetic field the heteronuclear coupling term can be truncated to

$$\mathscr{H}_{IS} = \sum_j d_j^{IS} I_{jz} S_z, \qquad [3]$$

where d_j^{IS} represents the heteronuclear coupling constant. Since $\mathscr{H}_S$ commutes with the rest of the Hamiltonian, it may be eliminated by transforming into an appropriate rotating frame. The matrix representation of the total Hamiltonian, written in the product base, is block diagonal. Each of the two blocks $\mathscr{H}^I_{1/2}$ and $\mathscr{H}^I_{-1/2}$, labeled by the corresponding eigenvalues of S_z, $m^S = \frac{1}{2}$ and $m^S = -\frac{1}{2}$, represent pure I-spin Hamiltonians which may be written as

$$\mathscr{H}^I_{\pm 1/2}(t) = \mathscr{H}_I \pm 1/2 \sum_j d_j^{IS} I_{jz} + \mathscr{H}_{II} + \mathscr{H}_{Irf}(t). \qquad [4]$$

Thus, the I spins represented by this Hamiltonian are shifted from the resonance of the isolated I-spin system by an amount $m^S d_j^{IS}$. Since only the S spins are observed, we can assume that the observable D commutes with all I-spin operators

$$[D, I_{j\alpha}] = 0,$$ [5]

where α runs over all degrees of freedom of the I spins. Furthermore we assume that the sampling interval of the signal is equal to the cycle time of the decoupling sequence τ_c. With the help of the commutation relation [5] we can write the observed signal as

$$\langle D \rangle (\tau_c(n+1)) = \mathrm{Tr}\{\rho(\tau_c(n+1))D\}$$

$$= \mathrm{Tr}\left\{ T \exp\left(-i \int_0^{\tau_c} \mathcal{H}_{IS}^S(t)dt\right) \rho(\tau_c n) \, T \exp\left(i \int_0^{\tau_c} \mathcal{H}_{IS}^S(t)dt\right)\right\},$$ [6]

where

$$\mathcal{H}_{IS}^S(t) = \sum_j d_j^{IS} \tilde{I}_{jz}^S(t) S_z$$ [7]

and

$$I_{jz}^S(t) = T e_{(-i \int_0^t (\mathcal{H}_I + \mathcal{H}_{II} + \mathcal{H}_{Irf}(t'))dt')} I_{jz} T e_{(i \int_0^t (\mathcal{H}_I + \mathcal{H}_{II} + \mathcal{H}_{Irf}(t'))dt')}.$$ [8]

Complete decoupling requires therefore that

$$\int_0^{\tau_c} I_{jz}^S(t)dt = 0,$$ [9]

where j runs over the indices of all I spins with a nonvanishing heteronuclear coupling constant. This means that a decoupling sequence must generate a propagator which is independent of the offsets of those I spins that are coupled to the S spins. This is entirely analogous to the Waugh decoupling criterion and is based purely on the effect of the pulse sequence on the isolated I-spin system. The residual splittings of the S-spin transition can actually be calculated if the average Hamiltonian for the I-spin system under the influence of the decoupling sequence is known as a function of the offsets of all I spins. This average Hamiltonian is defined as

$$e^{-i\bar{\mathcal{H}}^1 \tau_c} = T e^{-i \int_0^{\tau_c} (\mathcal{H}_I + \mathcal{H}_{II} + \mathcal{H}_{Irf}(t))dt}.$$ [10]

$\bar{\mathcal{H}}^1$ therefore represents the exact average Hamiltonian of the I-spin system and does not refer to an approximation like the Magnus expansion. Its dependence on the offsets of the I spins can be written as

$$e^{-i\bar{\mathcal{H}}^1(\delta_1, \ldots, \delta_j, \ldots)\tau_c} = T e^{-i \int_0^{\tau_c} (\mathcal{H}_I + \delta_1 I_{1z} + \cdots + \delta_j I_{jz} + \cdots + \mathcal{H}_{II} + \mathcal{H}_{Irf}(t))dt}.$$ [11]

We also introduce the shorthand notation

$$\Delta_r = (m_r^S d_1^{IS}, \ldots, m_r^S d_j^{IS}, \ldots)$$ [12]

for the N_I-dimensional vector Δ_r containing the additional offsets of the I spins caused by the heteronuclear coupling. Correspondingly, every eigenvalue E_t^I and eigenvector Ψ_t^I of $\bar{\mathscr{H}}^I$ will be regarded as a N_I-dimensional function of the offsets d_j^I. Because of the block diagonal form of the total Hamiltonian, the S-spin spectrum consists of all possible transitions $\Psi_{1/2}^S \Psi_t^I(\Delta_{1/2}) \rightarrow \Psi_{-1/2}^S \Psi_u^I(\Delta_{-1/2})$, where Ψ^S and Ψ^I denote the eigenstates of the isolated S- and I-spin systems. The frequencies are given by

$$f_{tu} = E_u^I(\Delta_{-1/2}) - E_t^I(\Delta_{1/2}) \tag{13}$$

and the relative intensities by

$$I_{tu} = (\Psi_t^I(\Delta_{1/2})|\Psi_u^I(\Delta_{-1/2})), \tag{14}$$

where (|) indicates the scalar product. The resulting S-spin spectrum consists therefore in general of $(2I + 1)^{2N_I}$ transitions which may be labeled with the corresponding states of the I-spin systems $\Psi_t^I \rightarrow \Psi_u^I$. Figure 1 shows the resulting multiplet structure for a spin $S = \frac{1}{2}$, coupled to a spin $I = 1$ which is subject to an MLEV-16 decoupling sequence. The three intense lines near the center of the spectrum correspond to $\Psi_t^I \rightarrow \Psi_t^I$ transitions. Their frequency is therefore determined by the difference between the values of the single energy level $E_t^I(\Delta_t)$, evaluated at the offsets $\Delta_{1/2}$ and $\Delta_{-1/2}$. The size of the residual couplings of these "parent lines" is therefore determined by the variation of the eigenvalues of the average I-spin Hamiltonian as a function of offset, and not by their absolute value. The less intense "satellites" are due to transitions $\Psi_t^I \rightarrow \Psi_u^I$ with $t \neq u$. Their frequencies are therefore determined by the total magnitude of the average Hamiltonian and their intensity by the variation of its eigenvectors as a function of offset. The satellites correspond to simultaneous transitions of the I- and S-spin system and their intensity is a measure of the mixing that occurs between the eigenstates of the I-spin average Hamiltonian when the offset is changed from $\Delta_{1/2}$ to $\Delta_{-1/2}$.

In the ideal case of an offset-independent average Hamiltonian the "parent lines" collapse into a single resonance and the intensity of the satellites vanishes. A full account of the decoupling performance of a pulse sequence therefore contains the
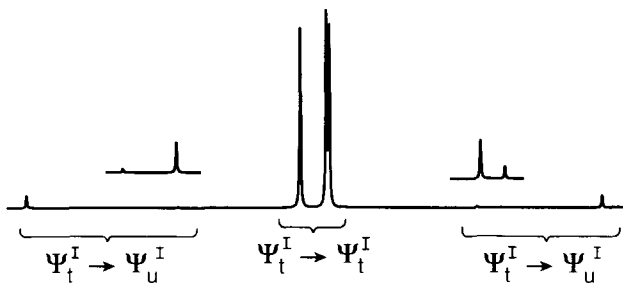

FIG. 1. Residual splittings and "satellites" in the spectrum of a spin $S = \frac{1}{2}$ coupled to a spin $I = 1$. The I spin is coupled to the electric field gradient and irradiated with an MLEV-16 sequence. The intense lines near the center of the spectrum correspond to transitions that leave the state of the I-spin system invariant, while the less intense satellites can be interpreted as combination lines.

offset dependence of the eigenvalues and eigenvectors of the I-spin average Hamiltonian. In practice, however, the loss of resolution and sensitivity associated with the residual splittings of the "parent lines" is usually considered more important than the intensity of the satellites. Pulse sequences that have a good decoupling performance and therefore generate eigenvalues with only small dependence on the offsets generally also generate eigenvectors that are relatively robust, leading to low intensity of the "satellites." Often the $\Psi_t^I \rightarrow \Psi_t^I$ transitions of the central multiplet are therefore the only ones that need to be taken into account. The remaining multiplet contains then $(2I + 1)^{N_I}$ resonance lines corresponding directly to the $(2I + 1)^{N_I}$ eigenvalues of $\mathscr{H}^I$. The measure for the decoupling performance of a pulse sequence over a certain bandwidth is then simply the difference between the minimum and maximum of each eigenvalue. These differences are equal to the biggest possible residual couplings in the S-spin spectrum, if the satellites can be neglected.

Figure 2 shows the bandwidth and maximum residual shifts for a one-dimensional cross section along $\delta_1^I = \delta_2^I$ through the two-dimensional energy surfaces of a system of two spins $I = \frac{1}{2}$ under the influence of a decoupling sequence. This figure also demonstrates that good decoupling performance does not automatically imply a small average Hamiltonian, but one that does not depend on offset. A nonvanishing offset independent average Hamiltonian that does not commute with the offset term may actually lead to an overall behavior that is less susceptible toward resonance offset since it partly quenches the offset term. As a consequence, the standard methods of generating improved pulse sequences by combining appropriate subcycles may fail in this case. While the procedure does reduce the size of the average Hamiltonian, it may make it at the same time more susceptible to resonance offset. The situation is shown schematically in Fig. 3.
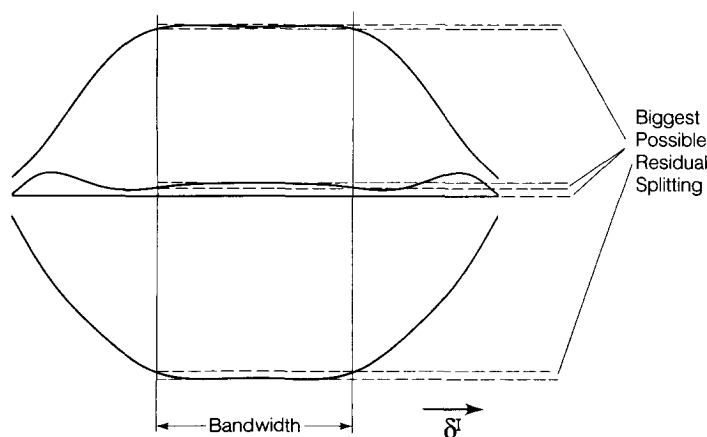

FIG. 2. Energy levels of the average Hamiltonian of a system of two equivalent spins $I = \frac{1}{2}$, subject to a COMARO-2 decoupling sequence (4), as a function of offset. The useful bandwidth of the sequence is given by the interval over which the energy levels are roughly independent of offset. The differences between the minimum and maximum of each level determines the largest possible residual splitting in the S spectrum. The straight line corresponds to the singlet state which is not affected by the offset.
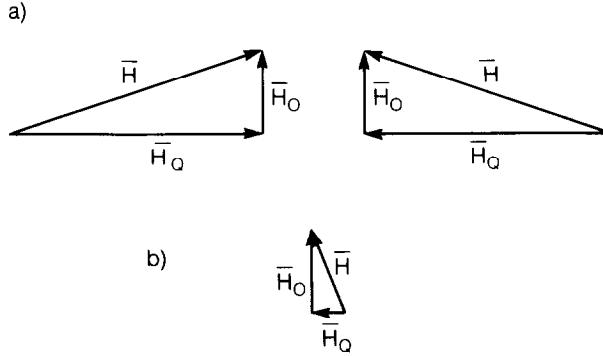
a)

b)

FIG. 3. Combination of subcycles to reduce error terms may actually decrease decoupling performance. (a) In the subcycles the offset term in the average Hamiltonian may be partly quenched by the orthogonal terms. (b) In the combined cycle, the orthogonal part has been greatly reduced and the offset term has become first order.

The cross section $\delta_1^I = \cdots = \delta_j^I = \cdots = \delta^I$ through the energy surfaces (see Fig. 2) contains all the necessary information to calculate decoupling performance and the residual couplings in the S spectrum if the I spins are equivalent, i.e., $d_1^{IS} = d_2^{IS} = \cdots = d^{IS}$. The resulting S spectrum for a certain coupling constant d^{IS} can be calculated as a function of decoupler offset δ by expanding the energy levels $E_t^I(\delta^I)$ in a Taylor series around δ

$$E_t^I(\delta^I) = E_t^I(\delta) + \sum_i (\delta^I - \delta) t_i, \qquad [15]$$

where

$$t_i = \frac{1}{i!} \frac{\partial^i E_t(\delta^I)}{(\partial \delta^I)^i} \bigg|_{\delta = \delta^I}. \qquad [16]$$

The shift of the corresponding S-spin resonance is given by

$$f_{tt} = E_t^I\left(\delta + \frac{d^{IS}}{2}\right) - E_t^I\left(\delta - \frac{d^{IS}}{2}\right) = 2\left[\frac{d^{IS}}{2} t_1 + \left(\frac{d^{IS}}{2}\right)^3 t_3 + \left(\frac{d^{IS}}{2}\right)^5 t_5 + \cdots\right] \qquad [17]$$

For small enough coupling constants d^{IS} the higher-order terms in this expansion can be neglected and the first derivative of the energy levels therefore represent scaling factors for the residual couplings in the S spectrum. Figure 4 shows an example for a spin system $I = 1$, $S = \frac{1}{2}$.

It is straightforward to recover the case $I = \frac{1}{2}$, $S = \frac{1}{2}$ treated by Waugh. The eigenvalues of the average Hamiltonian determine the average rotation frequency Ω, whose dependence on resonance offset, not whose total magnitude, causes the residual coupling in the S spectrum. The eigenvectors of the average Hamiltonian define the overall rotation axis which is assumed to be independent of offset in Waugh's treatment, thereby leading to vanishing intensity of the satellites.
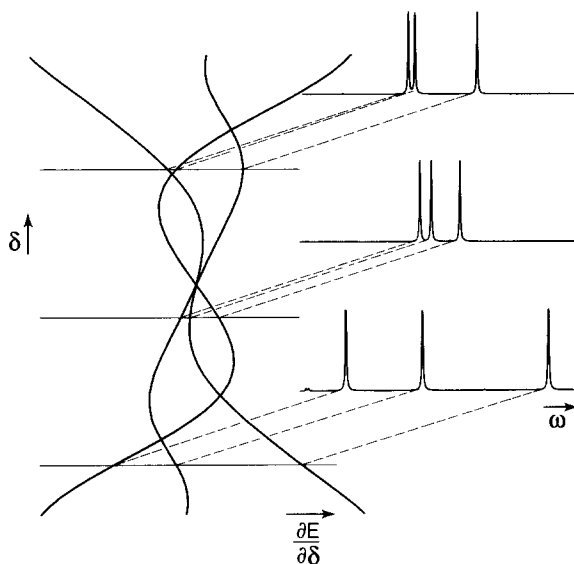
FIG. 4. For equivalent I spins, the derivative of the energy levels with respect to the offset represent scaling coefficients for the heteronuclear coupling constants if they are small. Here, the derivatives have been calculated for a spin $I = 1$, subject to a COMARO-2 sequence (4). The scaling coefficients are compared with spectra calculated for an IS system with $I = 1$, $S = \frac{1}{2}$ for three different decoupler frequency offsets.

SYSTEMS OF WEAKLY COUPLED S SPINS

If the observed spin system is more complex, the Hamiltonian of Eq. [2] must be expanded to include the additional interactions such as nuclear quadrupole interactions of the S spins and spin–spin couplings among them. However, in the high-field weak-coupling limit, these additional interactions all commute with the heteronuclear coupling and can therefore be taken into account separately. The resulting S spectrum may therefore be calculated in two steps: first the problem of the isolated S-spin system is solved and then each transition is coupled to the I-spin system separately. The Hamiltonian of the isolated S-spin system is still diagonal in the product base and accordingly each eigenstate Ψ_r^S can be labeled by the corresponding eigenvalues of the operators S_{kz}, m_{kr}^S. If the average Hamiltonian of the isolated I-spin system is coupled to state Ψ_r^S, the heteronuclear couplings generate a shift in the resonances of the I spins. The N_I-dimensional vectors containing the resonance shifts may be written as

$$\Delta_r = (\sum_k m_{kr}^S d_{1k}^{IS}, \ldots, \sum_k m_{kr}^S d_{jk}^{IS}, \ldots). \qquad [18]$$

A transition $\Psi_r^S \rightarrow \Psi_s^S$ of the isolated S-spin system is split into $(2I + 1)^{2N_I}$ transitions in the presence of the I spins. The transition $\Psi_r^S \Psi_t^I \rightarrow \Psi_s^S \Psi_u^I$ of the combined system is shifted from the position of the uncoupled resonance by

$$f_{rtsu} = E_u^I(\Delta_s) - E_t^I(\Delta_r) \qquad [19]$$

and has the relative intensity

$$I_{rtsu} = (\Psi_t^I(\Delta_r) | \Psi_u^I(\Delta_s)).$$ [20]

The average value of the magnetic quantum numbers of the S spins, $1/2(m_{ks}^S + m_{kr}^S)$ generates shifts of the I-spin resonances which can be included in a modified $\mathscr{H}_I$

$$\mathscr{H}_I^{rs} = \mathscr{H}_I + \sum_k \frac{1}{2}(m_{ks}^S + m_{kr}^S) \sum_j d_{jk}^{IS} I_{jz},$$ [21]

leaving a modified heteronuclear coupling which depends only on the difference

$$\mathscr{H}_{IS}^{rs} = S_z^{rs} \sum_j d_j^{rs} I_{jz}$$ [22]

with

$$d_j^{rs} = \sum_k (m_{ks}^S - m_{kr}^S) d_{jk}^{IS},$$ [23]

and S_z^{rs} represents a pseudo-spin $\frac{1}{2}$. The modification of $\mathscr{H}_I$ generates a shift in the resonance positions of the I spins and correspondingly increases the broadband requirements for the decoupling sequence. Since the different S-spin transitions generate different offsets of the I spins, the resulting residual couplings are different for each transition in the S-spin spectrum. The resulting S-spin spectrum can be calculated according to the treatment put forward in the second section for a single spin $S = \frac{1}{2}$ by considering each transition of the isolated S-spin system separately as a pseudo-spin $\frac{1}{2}$, using the modified $\mathscr{H}_I^{rs}$ and $\mathscr{H}_{IS}^{rs}$.

SUMMARY AND CONCLUSIONS

The theory of modulated heteronuclear decoupling can be generalized to arbitrary systems of I spins and weakly coupled S spins. A periodic irradiation scheme, applied to the I-spin system decouples them from another spin species S if it generates an average Hamiltonian $\bar{\mathscr{H}}^I$ that does not depend on the offsets of the various coupled I spins. It is not necessary that the average Hamiltonian be small. Offset dependence of the eigenvalues of $\bar{\mathscr{H}}^I$ leads to residual splittings in the S-spin spectrum while offset dependence of the eigenvectors leads to "satellites." The resulting spectrum of a weakly coupled system of S spins, coupled to an arbitrary system of I spins under the influence of a periodic decoupling sequence can be calculated in a two-step procedure where first the problem of the isolated S-spin system is solved and then every transition is coupled to the I-spin system. Each transition can be treated as a single spin $\frac{1}{2}$ if the offsets of the I spins are modified appropriately. In the linear regime, the first derivative of the energy levels as a function of offset corresponds to a scaling factor for the coupling constant. Derivative plots of the energy levels therefore are a useful tool for the analysis of decoupling performance as a function of offset. For larger couplings, outside the linear regime, higher derivatives must be taken into account. A complication which has not been taken into account is presented by strong coupling between the S spins. In this case the Hamiltonian for the isolated S-spin system does not commute with the heteronuclear coupling and the two-step procedure for the calculation of the

S-spin spectrum is no longer possible. The S_{kz} operators are no longer constants of the motion and accordingly the virtual offsets of the I spins, generated by the heteronuclear coupling, become time dependent, thereby interfering with the compensation scheme of the composite pulse sequence. An exact calculation of residual splittings in this case requires that the S-spin system be taken fully into account. However, pulse sequences that perform well for weakly coupled S-spin systems also yield good results in the strongly coupled case.

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